Quantum Gravitation and the Perihelion Anomaly.

J. N. KIDMAN (ALIAS : H. ASPDEN)
10 Birch House, Lingwood Close - Bassett, Southampton, England

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Consider the following hypothesis: If a particle of relatively small mass m is acted upon by a particle of relatively large mass M so as to be accelerated towards M at a rate f, then the force which M exerts on m is that applicable when m is a distance s further away from M, s being the distance $\frac{1}{2}ft^2$ corresponding to the acceleration f in the time t, where t is the time taken for a photon to travel from m to M and back to m.

This involves a retarded action of the force which M exerts on m, in as much as it takes time for the field system the adjust to the motion of m.

On the basis of this hypothesis we find that when m is in orbit about M and subject to mutual gravitational attraction then m describes an ellipse whose major axis advances about a focus in the direction of motion of m. This corresponds to the advance of perihelion for planetary motion. The equation of motion is identical to that deduced using the general theory of relativity.

Since the hypothesis could have quantum features, by virtue of its dependence upon a propagation mechanism similar to that of photons for establishing the interaction force, it may be that a link can be established between general relativity and quantum gravitation. The need for a new physics to account for such a link has recently been discussed by Ginzberg ([1]), who has, however, expressed the view that quantization of general relativity is unimportant under normal astronomical conditions. Nevertheless it may be that phenomena evident only in the astronomical situation, such as the anomalous motion of planetary perihelia, can lead us to the connection between the quantum world and the world of gravitation.

This paper serves solely to present the argument that the above hypothesis gives the same equation for the advance of perihelion of a planet as does that derived from general relativity. It is left to the reader to assess the relevance of this to the physical interpretation of the natural phenomena involved. The identity of the result with that obtained by Einstein is possibly significant in itself, because there are several theories of gravitation from which perihelion formulations are derived. A comparison of such theories has been presented by WHITROW and MORDUCH ([2]).

([1]) V. L. GINZBERG: *Quart. Journ. Roy. Astron. Soc.*, **16**, 265 (1975).
([2]) G. J. WHITROW and G. E. MORDUCH: *Nature*, **188**, 790 (1960).

To calculate the perihelion motion we apply Kepler's third law of planetary motion: the squares of the periodic times are proportional to the cubes of the mean distances of the planets from the Sun. An orbit period T is related to the distance R separating the two masses by the relation:

$$T \propto R^{\frac{3}{2}} . \tag{1}$$

There is no perihelion advance in the Newton-Kepler system due to two-body interaction. A radial perturbation has the same harmonic frequency as the orbital motion. There is perihelion advance if the radial perturbation has an oscillation period which is slower than the orbital period. Thus if (1) represents the orbital period and is modified due to the change of R to $R+s$ for radial oscillations, as results from the above hypothesis, we will expect a perihelion motion. The radial-oscillation period is $T(1+s/R)^{\frac{3}{2}}$, a fractional increase over T of $3s/2R$, if s is small relative to R. Now to find s we put $t = 2R/c$, where c is the speed of light. The acceleration f is v^2/R, where v is the orbital-speed component. T is then $2\pi R/v$. Since $s = \frac{1}{2}ft^2$, these data give $3s/2R = 3(v/c)^2$, which is the proportional rate of advance of perihelion. The advance in radians per revolution is 2π-times this quantity or, in terms of T,

$$\frac{24\pi^3}{T^2}\,\frac{R^2}{c^2} . \tag{2}$$

For elliptical motion in which eccentricity e is significant, the above retardation effect modifies Newton's law of gravitation:

$$m\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + mu = \frac{GMm}{h^2} \tag{3}$$

to the Einstein law of gravitation:

$$m\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + mu = \frac{GMm}{h^2}\left(1 + \frac{3h^2u^2}{c^2}\right) \tag{4}$$

and leads to a perihelion advance of

$$\frac{24\pi^3}{T^2}\,\frac{a^2}{c^2(1-e^2)} \tag{5}$$

in radians per revolution, presented here in the form used by Einstein [3] in his 1916 paper.

In these expressions $u = 1/R$, $h = vR$, G is the constant of gravitation, θ is the angular position of m in its elliptical orbit of major semi-axis a.

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[3] A. Einstein: *The Principle of Relativity* (New York, N. Y., 1952), p. 109. (This is collection of papers including Einstein's paper on *The Foundation of the General Theory of Relativity*, in *Ann. der Phys.*, **49** (1916).)