The Paradox of Constant Planetary Mass as Evidence of a Leptonic Lattice-Structured Vacuum State.

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Summary. – The constancy of velocity moment h in the equations of planetary motion has been well verified by their accuracy in accounting for perihelion advance. There is, however, an implicit paradox, because the conservation of angular momentum requires constancy of planetary mass, even though speed in orbit varies. One way of resolving this paradox appears in the conformable properties of a lattice-structured vacuum state of recent record, which provides also a quantum connection, in having given account for Planck's radiation law and a theoretical evaluation of the fine-structure constant in precise accord with its measured value.

Introduction. – Because there is identity of inertial and gravitational mass, it is invariable practice in authoritative treatments of planetary motion to formulate equations of planetary motion which apply to unit mass and involve only the solar mass M, the constant of gravitation G and the co-ordinates and speed of the planet in orbit.

From the methods of general relativity it is found that the planet moves under the action of a four-dimensional metric of space distorted by the presence of the solar mass M and that when the resulting orbit is translated into a nonrotating Euclidean three-dimensional space metric it has the form given by

$$(1)\qquad (\mathrm{d}u/\mathrm{d}\varphi)^2+u^2=(GM/h^2)^2(e^2-1)+2GMu/h^2+2GMu^3/c^2\,.$$

This ignores minor terms. The polar co-ordinates are u, φ, where u is the reciprocal distance between Sun and planet and φ is the angular position of the planet in orbit. The parameter h is a constant, as is e, and c is the speed of light.

The corresponding equation deduced from Newtonian principles is

$$(2)\qquad (\mathrm{d}u/\mathrm{d}\varphi)^2+u^2=(GM/h^2)^2(e^2-1)+2GMu/h^2\,,$$

where h is the moment of velocity of the planet in orbit and e is the eccentricity of the elliptical orbit represented by the motion.

It is a simple matter to verify that in the classical co-ordinate frame to which eqs. (1) and (2) apply, an equation of the general form

$$\mathrm{d}^2u/\mathrm{d}\varphi^2 + (1 + A)u = B + Cu^2 \tag{3}$$

will give a progressive advance of orbital perihelion through the angle $2\pi(BC - \frac{1}{2}A)$ radians per revolution. Then, since (1) reduces to this form, when differentiated with respect to φ and divided by $2(\mathrm{d}u/\mathrm{d}\varphi)$, with $A = 0$, $B = GM/h^2$ and $C = 3GM/c^2$, we find that the advance of perihelion is $6\pi(GM/hc)^2$ radians per revolution.

When h is taken as the velocity moment of the planet Mercury the advance of perihelion given by this expression is found to be about 42 seconds of arc per century, as verified by observation. In contrast, eq. (2) gives A and C both equal to zero and so indicates no advance of perihelion. For this reason, eq. (1) as derived by Einstein is the accepted equation of planetary motion.

This introduction is, of course, quite conventional. It serves to show the absolute reliance which the interpretation of general relativity places upon the validity of the relationship between eqs. (1) and (3) in the classical reference frame.

The paradox. – Confining attention to motion referenced in the classical frame, but governed by classical dynamics, we may write an equation relating rate of change of energy in a conservative exchange between gravitational potential energy E and the kinetic energy of a planetary mass m:

$$\mathrm{d}E/\mathrm{d}t + v_\theta \mathrm{d}(mv_\theta)/\mathrm{d}t + v_r \mathrm{d}(mv_r)/\mathrm{d}t = 0\,. \tag{4}$$

Here, t is time and v_θ, v_r are, respectively, the transverse and radial velocities of the planet in its orbital motion, for which $r = 1/u$.

If we multiply throughout by $\mathrm{d}t/\mathrm{d}r$, which is $1/v_r$, eq. (4) reduces to the form

$$-\mathrm{d}E/\mathrm{d}r = m(v_\theta^2/r) - \mathrm{d}(mv_r)/\mathrm{d}t\,, \tag{5}$$

provided, and only provided, $mv_\theta r$ is a constant. In other words, eqs. (4) and (5) are compatible only if mh is constant, where h is the velocity moment of the planet. Equation (4) is an expression ensuring energy conservation and eq. (5) is the familiar expression for force balance. Both equations hold even though m might vary in orbit, but then we must have variation of h and this is incompatible with the verified connection between eqs. (1) and (3). If h varies, then other terms will appear which affect A or C or both.

There is no problem if m is constant, even though speed in orbit varies owing to the elliptical nature of the orbit. However, when we consider the linear acceleration of the mass m to a velocity v and equate the energy element $vd(mv)$ with $c^2 d(m)$, as required to comply with the mass-equivalence of energy, we find that mass must increase with speed even in the classical space metric. Multiply both terms by m before integrating and we obtain

$$(mv)^2 = (mc)^2 + (m_0 c)^2\,, \tag{6}$$

which gives m equal to $m_0(1 - v^2/c^2)^{-\frac{1}{2}}$, where m_0 is the value of m with v zero.

This is a verified equation also, as based on observation referenced in the classical space frame of the laboratory observer. This presents a paradox. If mass must change with speed and angular momentum must be constant in orbit, then velocity moment h must change in the orbit. Whatever may be the reasons assuring that h is constant in the four-dimensional metric of space governing the general-relativity analysis, the value of h projected into the three-dimensional space frame in which observations are referenced cannot be deemed to remain constant, unless there are special boundary conditions hitherto not considered.

Perihelion motion for constant angular momentum. – To estimate the effect of a variable h, with mh constant, we write

$$1/h^2 = (1/h_0^2)(1 + v^2/c^2) = (1/h_0^2)(1 + h_0^2u^2/c^2)\,, \tag{7}$$

where h_0 is a new constant. This merely allows for the relativistic increase in mass with speed, where v is much less than c.

Replacing h^2 in (1) and supposing that e^2 is small in relation to unity, we then find that the appropriate equation of the form (3) has the values $A = 2(GM/h_0c)^2$, $B = GM/h_0^2$ and $C = 6GM/c^2$.

This involves a very significant adjustment to the perihelion advance. It becomes $10\pi(GM/h_0c)^2$ radians per revolution, which is 5/3 times the verified value deduced from Einstein's theory with h presumed constant.

Clearly, therefore, unless we are prepared to abandon the general-relativity result and look elsewhere for an explanation of perihelion anomaly, it is obligatory to admit that h must remain constant for planetary motion, notwithstanding variation of speed in orbit. The paradox cannot be avoided by appeal to the abstract properties of the special four-dimensional space metric, because the problem shows itself only in the ultimate equations derived from the theory of relativity in the form applicable to the observation frame of classical three-dimensional space. The perihelion advance was not deduced from eq. (1) directly. The equation has to be solved in order to determine the position of the perihelion and the solution involves eliminating the differential terms, a procedure which relies on the assumed constancy of h. The paradox is, therefore, quite serious, in view of the importance of the theory of general relativity.

Though the above is the author's opinion, the standard way of explaining the paradox is to affirm that h, as used in the general-relativity equation (1), though constant, is not the classical velocity moment of the planet, even though the equation as thus derived is applicable in the three-dimensional reference frame. To determine h in relation to the orbit as observed, it then suffices to identity it as the effective mean value of the classical velocity moment taken over the whole orbital period. In fact, however, the paradox and its solution are not explicitly mentioned in the textbook treatments of the subject [1-3], where eq. (1), once developed from the general-relativity metric used, is analysed as if it applies in a three-dimensional frame of reference. Also, there is something unsatisfactory about a theory which requires h to be constant at a value determined, not by the momentary condition of a mass moving in a gravitational field, but rather by the mean value of a quantity as integrated over a complete cycle of orbital motion. How is h then determined if the trajectory is not that of a regularly

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(1) H. A. WILSON: *Modern Physics* (Blackie, London, 1937), p. 381.
(2) V. FOCK: *The Theory of Space, Time and Gravitation* (Pergamon, Oxford, 1964), p. 219.
(3) J. EHLERS: *Relativity Theory and Astrophysics*. - I: *Relativity and Cosmology* (American Math. Soc., Providence, 1967), p. 90.

recurrent cyclic elliptical motion? It is submitted that the paradox remains and that some other solution has to be found.

Proposed solution to the paradox. – The easiest way of resolving the paradox involves appeal to experiment. If observation of the anomalous motion of the perihelion of the planet Mercury requires h to be constant, according to general relativity, then we take that as a valid proposition. We know that experiment indicating variation of mass with speed has only been performed for discrete fundamental particles which are known to move relative to the laboratory reference frame of the observer. Relativistic mass increase requires motion relative to the electromagnetic reference frame, so that the kinetic energy of the motion materializes as matter itself sharing the same motion. Suppose, however, that bulk matter in structured crystal form or aggregations of such matter or liquid or dense gaseous substance defines the electromagnetic reference frame. Then the motion of such matter will entail the motion of the electromagnetic reference frame and the kinetic energy of the motion may not then materialize to augment the mass property. Experiments on high-speed charged particles are of no relevance in testing this proposition. A planet would, however, then exhibit no relativistic mass increase, as required to resolve the paradox.

The most direct way of verifying the proposition just advanced is to investigate the nature of the electromagnetic reference frame, not as a medium regulating the speed of light, but rather as a medium in which charged-lepton pairs are created and annihilated and in which the universal quantum of Planck's radiation law is determined.

The vacuum field may be regarded as a sea of leptons comprising intermediate leptons (electrons, positrons and muons) permeated by a locally conformable lattice composed of light leptons, which define the electromagnetic reference frame (a frame shared by bulk matter), and a more tenuous population of heavy leptons (tau-particles, gravitons). When bulk matter moves through space the light-lepton lattice shares the motion. Where this vacuum lattice comes into collision with adjacent lattice the light leptons mutually annihilate to transfer their energy into the intermediate lepton form and so merge into a background from which the light leptons can reappear to create new lattice structure at boundaries of lattice separation. Momentum properties are always in balance and the vacuum will not intrude so as to affect normal mass properties of matter, but it will be a source of vacuum energy fluctuations as understood in quantum electrodynamics.

Kinetic energy of matter moving through the lattice will have an independence and will materialize as additional mass sharing the same motion. Kinetic energy of matter related to the coupled motion with the lattice will, of necessity, share the same lepton energy exchange as the lattice itself and be represented by intermediate leptons forming part of the sea of field energy which does not move with the lattice. The inertial and gravitational properties of these leptons forming the vacuum state are not in evidence in the interaction between matter and, in consequence, the value of h applicable to the motion of the planet can be said to be constant.

This is the author's proposal for resolving the paradox described in this paper. It is not a mere hypothesis devised specifically for this purpose. On the contrary, the leptonic vacuum having a lattice structure was conceived as a medium having a natural oscillation frequency at the resonance frequency for electron-positron creation with a structure determining the photon mechanism and a geometry from which the fine structure constant can be calculated. The most relevant reference is a very recent paper [4]

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[4] H. ASPDEN: *Phys. Lett. A*, **110**, 113 (1985).

in which the reciprocal of the fine-structure constant is shown to be 137.035 965, fully in accord with the measured value.

However, whether or not the author's explanation of the paradox finds favour, the paradox cannot be ignored and must be discussed in future texts on the subject, if general relativity is to stand as a viable account of gravitational phenomena.