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A Theory of Pion Lifetime.

H. Aspden

IBM United Kingdom Ltd., Hursley Laboratories - Winchester, England

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In a recent letter (1) it was shown how the neutron lifetime of some (918 ± 14) s could be explained to an accuracy within these limits of error. The principles applied were those of the author's lattice-structured vacuum theory (2,3). According to this theory, muon energy quanta, as a pair of migrant point charges, move at random within cubic cells of lattice dimension:

$$d = 72\pi e^2/mc^2 , \tag{1}$$

but with a characteristic frequency

$$\nu = mc^2/h . \tag{2}$$

Earlier, the model has been applied in explaining and evaluating the muon lifetime, giving a result in very good accord with observation (4). Note that e is the electron charge, m is the electron mass, c is the speed of light and h is Planck's constant.

The author is now able to report how this theory applies to the decay processes of the pion. The theoretical lifetime is found to be in full accord with measurement.

Let μ^+ and μ^- denote the positive and negative members of a muon pair, respectively. We regard the pion as having a spherical charge form, satisfying the basic Thomson formula used throughout the author's work and therefore having a radius given by

$$2e^2/3Mc^2 , \tag{3}$$

where M is the pion mass. Our object is then simply to determine the probability that the migrant muon charges will come close enough to the pion charge to satisfy a critical condition which triggers instability. We take this to arise when the muon charge overlaps the pion charge as the muon spreads from a point to assume the form set by the Thomson formula. The migration of the muon charge involves a process of expansion and decay from its point form following its repeated recreation at random positions at

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(1) H. Aspden: *Lett. Nuovo Cimento*, **31**, 383 (1981).

(2) H. Aspden and D. M. Eagles: *Phys. Lett. A*, **41**, 423 (1972).

(3) H. Aspden and D. M. Eagles: *Nuovo Cimento A*, **30**, 235 (1975).

(4) H. Aspden: *Physics Unified* (Southampton, 1980), p. 146.

the frequency ν. Until decay of the pion is triggered, we expect the combined energy of the pion, the muon and their mutual Coulomb interaction to be conserved. Then, to formulate the condition for pion decay, we consider a muon appearing as a point charge in the near vicinity of the pion and immediately adjusting to its spherical form set by (3), but with Mc^2 reduced by the amount of interaction energy. If this puts the muon charge into touching or overlapping relationship with the pion charge, then there will be instantaneous decay. The lifetime of the pion is then determined by the chance of this event occurring.

Suppose the pion charge is positive and denoted e and let x represent its fixed radius. Let the muon μ^+ appear at a distance y from the centre of the pion and adjust to a radius set by the Thomson formula (3). Instead of using mass energy Mc^2, now write P and Q as the energies of the pion and muon, respectively, in terms of the electron mass energy mc^2. Then the condition for instability, or touching relationship of the Thomson spheres, is simply

$$(P+Q)mc^2 = \frac{2}{3}\frac{e^2}{x} + \frac{2}{3}\frac{e^2}{(y-x)} + \frac{e^2}{y}\,. \tag{4}$$

Note that y must be greater than x. For the negative muon μ^- the above expression holds with the last term negative.

We seek to solve (4) to determine y. Then, by the same principles as were applied previously (^{1}), the pion lifetime τ^+ attributable to the effect of the migrant μ^+ is given by

$$\tau^+ = \frac{1}{\nu}\left(\frac{3d^3}{4\pi y^3}\right), \tag{5}$$

which, from (1), (2) and (3) is

$$\tau^+ = 3(108\pi N)^3 h/4\pi mc^2\,, \tag{6}$$

where

$$Nmc^2 = 2e^2/3y\,. \tag{7}$$

To solve (4), first write

$$Pmc^2 = 2e^2/3x\,. \tag{8}$$

Then cancel this term from both sides of eq. (4) and convert the equation, using (7) and (8), into the form

$$Q = N\left(\frac{P}{P-N} + \frac{3}{2}\right). \tag{9}$$

This can be solved, since it is a quadratic equation in N^2, to obtain

$$3N = (Q + 5P/2) \pm \sqrt{(Q+5P/2)^2 - 6PQ} \tag{10}$$

or, if we consider the negative-muon solution,

$$3N = -(Q - P/2) \pm \sqrt{(Q-P/2)^2 + 6PQ}\,. \tag{11}$$

Since y is greater than x, we can only have solutions for which N is less than P. Also, N must be positive. Thus, (10) gives $N = 72.3$ and (11) gives $N = 172$, with $P = 273$ and $Q = 207$, the known masses of the pion and muon.

The value of τ^+ is then found, from (6), to be $2.85 \cdot 10^{-8}$ s, since h/mc^2 is $8.09 \cdot 10^{-21}$ s. The value of the lifetime τ^-, attributable to the negative muon, is also found to be $38.4 \cdot 10^{-8}$ s. These two lifetimes combine according to the relationship

$$1/\tau = 1/\tau^+ + 1/\tau^- \,. \tag{12}$$

to give the mean value of lifetime τ. Thus, we may deduce a value of τ of $2.65 \cdot 10^{-8}$ s.

Supplementary to the decay effects of the muons, owing to their presence close to, but outside of, the pion charge sphere, there is the likely decay situation resulting from the change of the point charges appearing within the pion radius x. This is found by substituting P for N in (6) to obtain, for each muon, a further τ component value of $153 \cdot 10^{-8}$ s. This modifies the theoretical pion lifetime to $2.61 \cdot 10^{-8}$ s or $2.56 \cdot 10^{-8}$ s, according to whether one or both of the muons are effective, on entry into the pion charge, in causing its decay.

The measured lifetime of the pion is reported (5) as being $(2.551 \pm 0.026) \cdot 10^{-8}$ s and so we do have excellent accord between theory and observation.

Further research is now needed to extend the theory to other fundamental particle lifetimes. The neutron lifetime of the order of 10^3 s was explained by exploiting the dual probability of both the μ^+ and the μ^- acting in concert on separate quark elements of the neutron. The muon lifetime of the order of $2 \cdot 10^{-6}$ s was explained, as in the partial case above, by regarding the migrant muons μ^+ and μ^- as entering the charge body of the muon undergoing decay. In contrast, the pion decay has emerged as a lifetime of the order of $2.6 \cdot 10^{-8}$ s by considering the proximity effect of μ^+ and μ^- in triggering pion decay. We must therefore ask why there is no such effect in relation to muon decay. One can only speculate on this pending the outcome of further analysis on other particles. Possibly, a migrant muon can only induce decay of a particle of like energy by creating a situation not compatible with the energy conservation law and breaching an energy threshold. This was the basis on which the muon decay was evaluated (4). In the meantime, it is submitted that the ease with which the earlier theory has found application in explaining the pion lifetime is a most encouraging feature, enhancing the credibility of the author's theoretical method.

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(5) R. D. Hill: *Condon and Odishaw Handbook of Physics* (New York, N. Y., 1967), p. 9.